PCAT Test Blueprint and Sample Items

Biological Processes

Biological Processes Objectives	
B1. General Biology	**B2. Microbiology**
A. Cellular and Molecular Biology	**A. Microorganisms**
1. Structure and functions of cells	**B. Infectious Diseases & Prevention**
2. Gene expression	**C. Microbial Ecology**
3. Cell division and growth	**D. Medical Microbiology**
4. Energy transformations	**E. Immunity**
5. Metabolism	**B3. Human Anatomy and Physiology**
B. Diversity of Life Forms	**A. Structure**
1. Genetics	1. Cells
F. Health	2. Tissues
1. Nutrition	3. Organs
2. Diseases	**B. Systems**
3. Drugs	1. Skeletal/muscular/nervous
	2. Circulatory/respiratory
	3. Excretory/digestive
	4. Endocrine/reproductive
	5. Integumentary/immune

Examples of Biological Processes Items

Biological Processes items may be presented either standing alone (Biology Example 1–4) or associated with a short passage (Biology Example 5). Stand-alone items can be answered independent of any passage or other item, while items associated with a passage will require understanding part or all of the passage in order to answer them correctly.

∉ Each Biological Processes item stem will be in the form of a question (followed by a question mark) or in the form of an incomplete sentence that requires completion (with no end punctuation).

• Options may contain more than one concept or piece of information but each one will plausibly relate to the stem.

In the following sample Biological Processes items, the correct answer for each item is followed by and asterisk (*), and each answer option is followed by a bracketed explanation for why it is correct (CA) or incorrect (neither of which will appear in the actual test).

Biological Processes Example 1

Content Objective: B2E

Any molecular configuration that triggers the formation of the lymphocyte immune response and is itself the target is called a(n)

A. leukocyte.
[A leukocyte is a white blood cell and not the target of an infection from a foreign molecule.]

B. eosinophil.
[Eosinophil is a type of white blood cell that is involved in the immune response to parasitic infections or allergic reactions and not the target of infection from a foreign molecule.]

C. immunoglobulin.
[Immunoglobulins are antibodies formed by B cells and not the targets of an infection from a foreign molecule.]

D. antigen. *
[CA: *Cpʋki gp* is the correct term for anything that is the target of the immune response, causing production of antibodies by the living organism. Antigens can include foreign pollen, bacteria, viruses, proteins, and some other materials.]

Biological Processes Example 2

Content Objective: B1F2

Emphysema is a lung disease that impairs the exchange of oxygen and carbon dioxide, resulting in which condition?

A. Respiratory alkalosis
[Respiratory alkalosis is due to alveolar hyperventilation leading to decreased plasma carbon dioxide concentration. It develops when the lungs remove more carbon dioxide than is produced in the tissues. It is a common finding in patients receiving medical ventilation. Emphysema results in a decreased expulsion of carbon dioxide.]

B. Metabolic alkalosis
[This conditions results from an altered metabolism. A decreased hydrogen ion concentration results in increased bicarbonate and carbon dioxide concentrations. It occurs most commonly when a person has been vomiting profusely. Emphysema results in a decreased expulsion of carbon dioxide.]

C. Respiratory acidosis *
[CA: Respiratory acidosis is a clinical disturbance that is due to alveolar hypoventilation. It results in low blood pH due to decreased clearance of carbon dioxide by the lungs. This condition occurs in emphysema as exhalation becomes insufficient.]

D. Metabolic acidosis
[Metabolic acidosis is a condition in which the blood pH is low due to increased production of hydrogen ions by the body or the inability of the body to form bicarbonate in the kidney. Emphysema results in a decreased expulsion of carbon dioxide.]

Biological Processes Example 3

Content Objective: B1B1

A child is born with trisomy 21. DNA testing using a polymorphic DNA marker for chromosome 21 revealed the pattern exhibited in the Southern blot below.

Father   Mother   Child
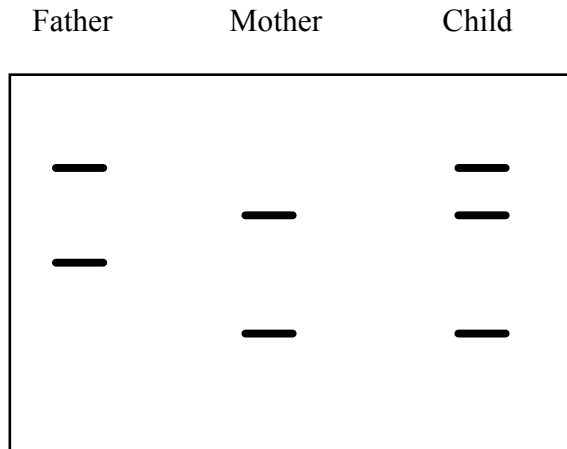
At which meiotic division did nondisjunction occur?

A. Maternal meiotic division I *

[CA: Nondisjunction is an error that can occur during meiosis or mitosis, causing the daughter cells to have too many or too few chromosomes. Because the child has two maternal alleles that are not identical and one paternal allele, nondisjunction occurred at this stage of division.]

B. Paternal meiotic division I

[If nondisjunction occurred here, the child would have two paternal alleles and one maternal allele.]

C. Maternal meiotic division II

[If nondisjunction had occurred here, the maternal contribution would have been either no allele or two of the same kind.]

D. Paternal meiotic division II

[If nondisjunction had occurred here, the paternal contribution would have been either no allele or two of the same kind. Since there are two that appear to be of maternal origin, there is only one of paternal origin.]

Biological Processes Example 4

Content Objective: B3B3

Which sphincter in the gastrointestinal tract controls movement of chyme from the stomach into the small intestine?

A. Cardiac

[The cardiac sphincter (lower esophageal sphincter) is located at the junction of the esophagus and the stomach.]

B. Pyloric *

[CA: The pyloric sphincter separates the stomach from the duodenum, the first division of the small intestine. The pyloric sphincter normally is in a contracted state, and retains food within the stomach until it has been broken down mechanically and chemically into chyme that can be passed into the duodenum of the small intestine.]

C. Ileocecal

[The ileocecal sphincter is located at the junction of the small intestine (ileum) and the large intestine.]

D. Anal

[The anal sphincter is located at the end of the rectum.]

Biological Processes Example 5: Passage and Accompanying Item

Chronic Stress and Immunity

Stress is an intrinsic part of life for most organisms, and dealing successfully with stressors enables survival. Early studies in rats showed that acute stress results in a redistribution of leukocytes from the blood to other organs such as skin, lymph nodes, and bone marrow, and that adrenal stress hormones are the major mediators of this response. Because the skin is one of the targets to which leukocytes traffic during stress, researchers hypothesized that such a leukocyte redistribution may increase immune surveillance and consequently enhance immune function if skin is exposed to an antigen after acute stress.

To test this hypothesis, researchers examined the effects of acute stress on skin immunity by exposing rats to containment in a restraining cage (see partial results below in Table 1). Physical restraint activates the sympathetic nervous system and the hypothalamic-pituitary-adrenal axis, and results in the activation of adrenal steroid receptors throughout the body. Results showed larger numbers of leukocytes in the skin of stressed animals both before and after exposure to an antigen. In contrast to the immunoenhancing effects of acute stress, researchers found that chronic stress significantly suppressed the immune response. An adrenalectomy eliminated the stress-induced enhancement of skin immune response.

Table 1: Physiological Change from Baseline in Response to Stimuli

Stimulus	Heart Rate	Bronchiole Diameter	Bone Marrow Leukocytes
A	Increase	Decrease	Increase
B	Increase	Increase	Increase
C	Decrease	Increase	Decrease
D	Decrease	Decrease	Decrease

Biological Processes Example 5

Objective: B3B3

If the study described in the passage had examined the effects of an acute stressor on digestion, the information in the passage would best support the prediction that

A. peristalsis would be increased.
[Sympathetic activation results in decreased rather than increased peristalsis.]

B. intestinal motility would be decreased. *
[CA: According to the passage, an acute stressor results in activation of the sympathetic nervous system. Sympathetic effects on digestion include decreased intestinal motility and peristalsis, decreased salivation, and increased anal sphincter tone.]

C. salivation would be increased.
[Sympathetic activation results in decreased rather than increased salivation.]

D. anal sphincter tone would be decreased.
[Sympathetic activation results in increased rather than decreased anal sphincter tone.]

Chemical Processes

Chemical Processes Objectives	
C2. General Chemistry	**C3. Organic Chemistry**
A. Atomic Theory	**A. Structure and Properties**
1. Structure	1. Structural formulas and bonding
2. Ions	2. Properties of organic compounds
3. Periodicity	**B. Reactions of Organic Compounds**
B. Chemical Bonding	1. Oxidation–reduction reactions
1. Nomenclature/formulas	2. Hydration and dehydration
2. Bonding	3. Hydrolysis
C. Reactions and Reaction Mechanisms	4. Addition/substitution/elimination
1. Types of reactions	**C4. Basic Biochemistry Processes**
2. Balancing equations	**A.** DNA and RNA
3. Equilibrium	**B**. Lipids
4. Stoichiometry	**C**. Proteins
D. Kinetic Theory	
1. States of matter	
2. Gas laws	
3. Causes and effects of changes in states	
E. Solutions	
1. Concentration (pH)	
2. Solubility	
3. Acid–base theories	
G. Nuclear Chemistry: Radioisotopes	

Examples of Chemical Processes Items

- Chemical Processes items may be presented either standing alone (Chemical Processes Example 1–4) or associated with a short passage (Chemical Processes Example 5). Stand-alone items can be answered independent of any passage or other item, while items associated with a passage will require understanding part or all of the passage in order to answer them correctly.

- Each Chemical Processes item stem will be in the form of a question (followed by a question mark) or in the form of an incomplete sentence that requires completion (with no end punctuation).

- Options may contain more than one concept or piece of information but each one will plausibly relate to the stem.

In the following sample Chemical Processes items, the correct answer for each item is followed by an asterisk (*), and each answer option is followed by a bracketed explanation for why it is correct (CA) or incorrect (neither of which will appear in the actual test).

Chemical Processes Example 1

Content Objective: C2D3

A student has a sample of a gas in a glass container with a movable piston. In order to convert this gas to a liquid, the student should

A. increase the inside pressure by decreasing the volume, lowering temperature as needed. *
[CA: These conditions will lead to the reduction in volume and kinetic energy needed to convert the sample to a liquid phase.]

B. transfer the gas to a container of larger volume.
[This will increase the volume and not move the sample toward the liquid state.]

C. increase the inside temperature by increasing the volume, decreasing pressure as needed.
[This will cause an increase in volume and kinetic energy and will not move the sample toward the liquid state.]

D. transfer the gas to a container of different shape.
[This will not affect the interactions between molecules.]

Chemical Processes Example 2

Content Objective: C2A3

The diagram below shows part of the periodic table with the first ionization energy given for arsenic (As).

Al	Si	P	S
Ga	Ge	As 947 kJ/ mol	Se
In	Sn	Sb	Te

Which elements are most likely to have an ionization energy that is 1,000 kilojoules/mole (kJ/mol) or higher?

A. Aluminum (Al) and silicon (Si)

[Though Al and Si are above As, they are both to the left and are not as likely to have IE's $\geq$ 1,000 kJ/mol; Note: IE_{Al} = 577 kJ/mol and IE_{Si} = 786 kJ/mol.]

B. Indium (In) and tin (Sn)

[Both In and Sn are below and to the left of As, so IE < 1000 kJ/mol; Note: IE_{In} = 558 kJ/mol and IE_{Sn} = 709 kJ/mol.]

C. Antimony (Sb) and tellurium (Te)

[Antimony is below As, so IE < 1000 kJ/mol; and though Te is to the right, it is lower than As and not as likely to have an IE $\geq$ 1,000 kJ/mol; Note: IE_{Sb} = 834 kJ/mol and IE_{Te} = 869 kJ/mol.]

D. Phosphorus (P) and sulfur (S) *

[CA: IE generally increases from left to right and going up the periodic table. Since P is above and S is above and to right of As, IE's are most likely $\geq$ 1,000 kJ/mol; Note: IE_P = 1,012 kJ/mol and IE_S = 1,000 kJ/mol.]

Chemical Processes Example 3

Content Objective: C3B2

Which of the following products is formed when ethyne reacts with water and H_2SO_4 in the presence of mercuric sulfate $HgSO_4$ at 60 ºC?

A. CH_2CH_2
[This would require a different set of reactants, namely, H_2 and a Lindlar catalyst.]

B. CH_3CHO *
[CA: The hydration of ethyne breaks the triple bond and produces acetylaldehyde (ethanal).]

C. CH_3CH_3
[This would require a different set of reactants, namely, H_2 and a Lindlar catalyst.]

D. CH_3CH_2OH
[The candidate may have confused alkenes, which give alcohol products, with alkynes.]

Chemical Processes Example 4

Content Objective: C4C

The loss of a secondary, tertiary, or quaternary protein structure due to the disruption of noncovalent interactions and/or disulfide bonds that leaves the primary structure intact is known as

A. esterification.
[Esterification is a reaction that usually involves the combination of an acid and an alcohol and leads to the production of an ether.]

B. denaturation. *
[CA: Denaturation is a biochemical process that involves an unfolding of a protein's secondary, tertiary, or quaternary structure but does not affect the primary structure. Denatured proteins can exhibit a wide list of changes, such as a decrease in solubility.]

C. saponification.
[Saponification is a reaction that involves the alkaline hydrolysis of an ester to produce soap and glycerin.]

D. dehydration.
[Dehydration is a reaction that usually involves the loss of water molecules.]

Chemical Processes Example 5: Passage and Accompanying Item

Chemical Defense Mechanisms

Biological systems quite frequently utilize basic organic chemistry reactions and pathways to perform a variety of tasks. One common example of this can be seen in the chemical defense mechanisms that many insects and plants use to protect themselves when threatened or damaged. For example, the millipede *Apheloria corrugate* stores a cyanohydrin known as mandelonitrile in specialized glands, which it uses when threatened to undergo an enzyme-catalyzed reaction to produce hydrogen cyanide. Hydrogen cyanide is highly toxic to the predator, and an adult *Apheloria corrugate* can produce up to 0.6 milligrams of HCN, approximately 6 times the lethal dose for a 25-gram mouse. Plants in the *Cruciferae* family (e.g., broccoli, brussels sprouts, cabbage, and mustard greens) utilize a similar defense. When leaves of these plants become damaged, stored glucosinolates undergo an enzyme-catalyzed reaction to produce isothiocyanates and/or nitriles, as shown in Figure 1. Interestingly, the production of isothiocyanates is not highly toxic to higher order predators and has actually been found to have anti-cancer properties in humans.

Figure 1: Reaction Products of the Enzyme Catalyzed Reaction of Glucosinolates
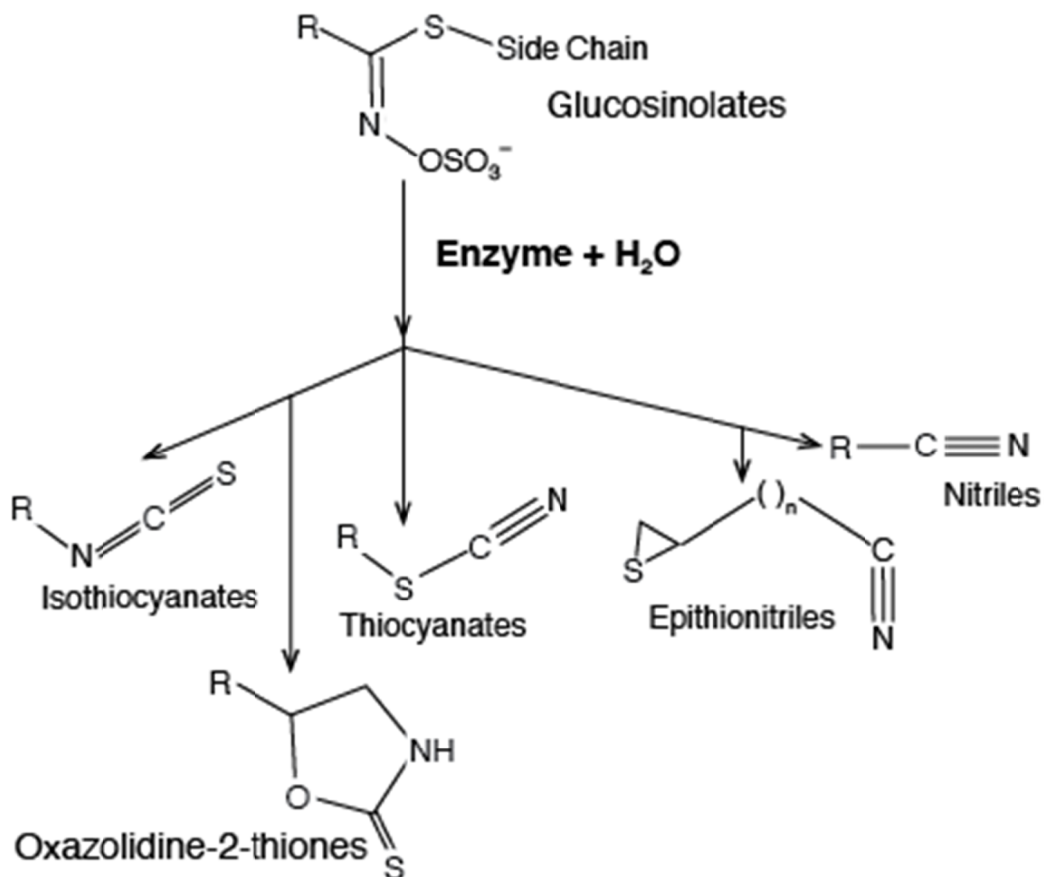
Chemical Processes Example 5

Objective: C3B3

The enzyme-catalyzed reaction shown in Figure 1 is best classified as a/an

A. hydrolysis. *
[CA: The reaction described is a hydrolysis reaction. Myrosinase acts as the enzyme catalyst between the glucosinolate and water, and hydrolysis occurs when a bond is cleaved by the addition of a water molecule.]

B. nucleophilic addition.
[A nucleophilic addition reaction occurs when a nucleophile adds to an electrophilic carbonyl group. The reaction shown in Figure 1 does not involve a carbonyl group.]

C. oxidation.
[Oxidation reactions typically require the formation of a C-O or C-N bond with the breaking of a C-H bond, thereby decreasing the electron ownership of the carbon group. In this particular example, the electron ownership of the C-N group does not change and no new C-O or C-N bonds are formed.]

D. reduction.
[In reduction reactions, electron ownership will increase for a carbon atom by bond formation between carbon and a less electronegative atom. In this particular example, the carbon atom does not break a bond between C-N to form C-H.]

Critical Reading

Critical Reading Content Objectives
R1. Comprehension (recognition, understanding)
A. Words in Context (defining a term used in the passage)
B. Main Ideas (identifying or inferring the main idea of a paragraph or group of paragraphs)
C. Supporting Details (identifying facts or ideas explicitly stated in the passage)
D. Drawing Conclusions (making inferences from statements in the passage)
R2. Analysis (inference, interpretation)
A. Relationships Between Ideas (identifying relationships between ideas in different parts of a paragraph or in different paragraphs)
B. Author's Purpose (inferring the author's purpose for writing the entire passage or for including a statement in part of the passage)
C. Author's Tone (inferring author's attitude in the entire passage or in a specific statement in part of the passage)
D. Facts/Opinions (distinguishing between statements of fact and expressions of opinion)
E. Rhetorical Strategies (identifying methods used by the author for effect, to persuade, or to make a point)
R3. Evaluation (reasoned judgment)
A. Bias (inferring author's viewpoint, preference, or position in entire passage or in a specific statement in part of the passage)
B. Support in an Argument (evaluating the effectiveness of elements of support used by the author in the passage)
C. Author's Conclusion/Thesis (identifying or inferring the author's overall point in the passage, or evaluating how well the author's overall point follows from the support provided)

Examples of a Critical Reading Passage and Items

- Each PCAT Critical Reading passage will address historical or contemporary social, cultural, ethical, political, or technical issues related to the applied or basic natural sciences, the social sciences, or the humanities.

- Passages may be informative, persuasive, or speculative in purpose or may be practical or theoretical in nature.

- Each reading item stem will clearly relate to the reading passage and will require understanding part or all of the passage in order to answer correctly.

- Each item stem may refer to a specific word, phrase, sentence, paragraph, or section of the passage or may refer to the passage as a whole.

Following the sample Critical Reading passage, the correct answer for each sample item is followed by an asterisk (*), and each answer option is followed by a bracketed explanation for why it is correct (CA) or incorrect (neither of which will appear in the actual test).

Critical Reading Passage Example

Low-Calorie Diet and Aging

1   An interesting question has emerged from recent research involving calorie restriction and aging: Can a low-calorie diet help people live longer? Studies involving animals such as mice, worms, and fish have shown that eating a very low-calorie diet can prevent such ailments as heart disease, diabetes, and some cancers and can actually extend life spans. The reasons for these results are not adequately understood, and scientists are uncertain whether the same results can occur in humans. However, some promising research findings are beginning to appear.

2   In studies conducted at the University of California, Riverside (UCR), researchers have found that by decreasing the intake of calories in laboratory mice, regardless of the time period involved, most diseases caused by aging are prevented. Based on findings from their most recent studies, these researchers suggest that it is possible to help avoid heart disease, cancer, and diabetes simply by restricting caloric intake for as short a time as four weeks. The UCR researches have also found that a chemical change takes place in the body through calorie restriction, and suggest that it is possible to duplicate this change through the use of pharmaceuticals.

3   Research is also beginning to reveal some interesting findings involving low-calorie diets and humans. Several studies have been conducted within human populations in Okinawa, Japan. Results of these studies have shown that individuals under the age of sixty show very little serious disease, and that there are forty times as many residents of Okinawa living 100 years or longer than in more northeastern areas of Japan. Studies have concluded that a lower calorie intake appears to be the main difference in the eating habits of Okinawans.

4   In findings published in 2006, researchers at the Pennington Biomedical Center at Louisiana State University reported on a study involving calorie restriction with forty-eight overweight humans. Though considered preliminary, the results of this study suggested that people on a six-month calorie-restricted diet showed several improvements in health markers associated with aging. In this study, one group of twelve individuals followed a diet with a 25 percent calorie restriction from the usual daily calorie amount. Another group combined a 12.5 percent increase in exercise with a 12.5 percent reduction in calorie intake. A third group followed an 890-calorie liquid diet for three months, followed by a weight-maintenance diet. A control group attempted

to merely maintain steady weight. While no changes were observed in the control group, the results of this study suggested that individuals who followed a strict low-calorie diet showed decreased amounts of age-related DNA damage when compared to initial levels, as well as decreased insulin levels and body temperature, both of which are considered signs of longevity.

5   The findings of studies such as this and others conducted more recently, including several funded by the U.S Department of Health and Human Services National Institute on Aging (NIA), suggest that the effects of a low-calorie diet on the aging process appear quite positive. In fact in one NIA-funded study reported in 2015, researchers concluded that calorie restriction with individuals who were not overweight positively influenced several key indicators associated with a longer life span. An especially interesting possibility suggested by some of these studies involves developing pharmaceuticals that may duplicate the chemical changes produced in the body from calorie restriction. Although more research is clearly needed to confirm preliminary findings, promising results so far suggest that following a calorie-restricted diet can very possibly help us avoid many of the diseases that now plague us as we grow older, and thus lead to longer, healthier lives.

Critical Reading Example Item 1

Content Objective: R1B

What main point from the passage is supported by the suggestion made by UCR researchers referred to in the second paragraph that "it is possible to help avoid heart disease, cancer, and diabetes simply by restricting caloric intake for as short a time as four weeks."

A. It seems that heart disease, cancer, and diabetes are all directly tied to diet.
[The author never states this unequivocally but simply as a growing theory.]

B. Improvements are often seen for patients with chronic diseases in a relatively short amount of time.
[The four-week time period is referring to the amount of time calories were restricted, not diseases.]
"
C. Preliminary research indicates that major diseases may be affected by calorie intake. *
[CA: The author is showing that early research had found correlations between caloric intake and incidence of major diseases.]
"
D. Many serious diseases can be avoided if a person makes sufficient effort to avoid them.
[This is never stated or implied anywhere in the passage.]

Critical Reading Example Item 2

Content Objective: R2D

Which words from the passage reflect the author's opinion?

A. ". . . recent research . . ." (Paragraph 1)
[Neither word implies an opinion, since *t gegpv* merely makes a time reference.]

B. ". . . adequately understood . . ." (Paragraph 1)
[Though the word *cf gs wc vgrf* can show opinion, in this context it seems to refer to how researchers assess the situation, rather than to how he author sees it.]

C. ". . . considered preliminary . . ." (Paragraph 4)
[The word *rt grko kpct{* could imply opinion, but in this context the word refers to how the Pennington Biomedical Research Center researchers consider their findings, not to how the author sees them.]

D. ". . . quite positive." (Paragraph 5) *
[CA: In this context, the word *s wkvg* implies a judgment concerning the degree to which the author considers the results to be *r qukkxg*, which also suggests the author's view.]

Critical Reading Example Item 3

Content Objective: R2E

Which strategy does the author use throughout the passage to support the overall thesis?

A. Adding personal anecdotes to highlight the best way to restrict calories
[There are no personal anecdotes, or stories based on personal experience, in the passage.]

B. Referring to respected publications for additional facts and statistics
[Even though study results are mentioned that have presumably been published, there are no references to the actual names of the publications in the passage.]

C. Listing results from prestigious studies to add credibility *
[CA: The author refers to studies in paragraphs 2, 4, and 5 that are associated with respected institutions in support of the main point.]

D. Repeating key words to link diverse and unique ideas together
[There is no obvious repetition of this sort and no attempt to represent diverse ideas in the passage.]

Critical Reading Example Item 4

Content Objective: R3B

Which point from the passage best supports the author's suggestion that a low-calorie diet may be able to extend the average life span?

A. Research subjects on a calorie-restricted diet showed decreased amounts of age-related DNA damage. *
[CA: This is suggested near the end of paragraph 4 and shows that calorie restriction does affect the basic DNA, which provides strong support for the author's main point in this passage.]

B. Many Okinawans live to be more than 100 years old.
[Although this statement made in paragraph 3 may be true, it is not itself evidence but is the kind of statement that other evidence in the passage related to a low-calorie diet is meant to support.]

C. Studies conducted on mice, worms, and fish have shown that calorie reduction prevents age-related diseases.
[The fact that positive changes occur in various animals (mentioned in paragraphs 1 and 2) is suggestive only and provides no convincing evidence for the benefits to humans of calorie reduction.]

D. A chemical change takes place in the body when calories are sufficiently restricted.
[Chemical changes are mentioned in paragraph 2, but the fact that a chemical change occurs is not explicitly connected to longevity in such as way as to make this fact as strong a support as other evidence presented elsewhere in the passage.]

Quantitative Reasoning

Quantitative Reasoning Content Objectives
Q1. Basic Math
A. Fractions, Percentages, & Decimals
B. Unit Conversions
C. Log Base 10
Q3. Algebra
G. Expressions, Equations, and Inequalities
1. Evaluate algebraic expressions for given values
2. Represent verbal quantitative situations as algebraic expressions or equations
3. Solve problems using linear equations and inequalities
4. Solve problems using equations and inequalities involving absolute value
5. Solve problems using equations and inequalities involving rational expressions
6. Solve quadratic equations and inequalities
7. Solve equations and inequalities involving 1 or 2 radicals
8. Solve systems of equations or inequalities involving 2 or 3 variables
I. Functions
1. Perform algebraic operations on functions
2. Determine compositions of functions
3. Determine inverses of functions
4. Determine and use maximum and minimum points
Q4. Probability & Statistics
A. Measures of Central Tendency
B. Variation
C. Graphical
D. Probability
E. Statistical Concepts

Quantitative Reasoning Content Objectives *(continued)*

Q5. Precalculus

A. Functions

    1. Graph and identify domains, ranges, intercepts, and zeros of exponential functions

    2. Logarithms (natural or other base with multiple operations)

    3. Solve problems related to exponential and logarithmic functions

    4. Graph and identify domains, ranges, intercepts, zeros, and inverses of the circular functions

    5. Perform algebraic operations on functions

    6. Identify and use composite functions

B. Complex Numbers

C. Vectors

    1. Add vectors graphically and algebraically

    2. Perform scalar multiplications

    3. Represent and/or recognize vector equations of lines and planes

Q6. Calculus

A. Limits (Find: Limits of functions, One-sided limits, Infinite limits)

B. Continuity (Interpret graphs of continuous and discontinuous functions)

C. Derivatives

    1. Find derivatives of algebraic functions by means of the Sum and product, Power rule, apply the Mean Value Theorem

    2. Use the Chain Rule to find derivatives of composite functions

    3. Solve problems by differentiation (e.g., velocity and acceleration)

    4. Use and/or interpret derivative tests to find extrema, points of inflection, intervals

    5. Interpret and/or use the derivatives of circular functions and their inverses

    6. Interpret and/or use the derivatives of transcendental functions

    7. Determine the derivatives of composite functions involving the circular and transcendental functions

    8. Use implicit differentiation

    9. Determine related rates

D. Integrals

    1. Find antiderivatives, and interpret C

    2. Understand and use sigma notation for simplifying sums

    3. Approximate areas bounded by curves

E. Integration

Examples of Quantitative Reasoning Items

- Each Quantitative Reasoning item stem will be in the form of a question (followed by a question mark), in the form of an incomplete sentence that requires completion (with no end punctuation), or in the form of an incomplete statement that ends in an equals sign (=).

- Options may contain more than one concept or piece of information but each one will plausibly relate to the stem.

In the following sample Quantitative Reasoning items, the correct answer for each item is followed by an asterisk (*), and each answer option is followed by a bracketed explanation for why it is correct (CA) or incorrect (neither of which will appear in the actual test).

Quantitative Reasoning Example 1

Content Objective: Q3G7

Let c be a constant. The lines $2x - y = 5$ and $x - y = c$ intersect in a unique point (r, s) in the coordinate plane. In which way does increasing the value of c affect the values of r and s?

A. The values of r and s both decrease. *
[CA: Solve the system by eliminating y; subtract the second equation from the first equation to get $x = 5 - c$; then substitute into the equation $x - y = c$ to get $5 - c - y = c$; $y = 5 - 2c$. So, $r = 5 - c$; $s = 5 - 2c$; both r and s decrease with c because the coefficient of c is negative in the expressions for r and s.]

B. The value of r increases; the value of s decreases.
[The candidate chooses the wrong direction of change for r.]

C. The values of r and s both increase.
[The candidate chooses the wrong directions of change for both r and s.]

D. The value of r decreases; the value of s increases.
[The candidate chooses the wrong direction of change for s.]

Quantitative Reasoning Example 2

Content Objective: Q1A

A student folded a square sheet of paper in half four times and then unfolded it. The dashed lines in the figure below show where the student folded the sheet of paper.
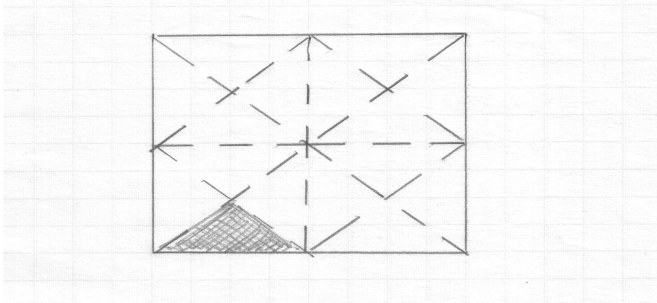
Before the student unfolded the paper, it looked like the shaded triangle. Which is closest to the percent of the sheet of paper that is the shaded triangle?

A. 4%
[The candidate thinks that the paper was folded in half four times and that this equals 4%.]

B. 6% *

[CA: Solve this problem by first determining that the shaded area is $\dfrac{1}{16}$ of the sheet of paper:

$\dfrac{1}{16} = 0.0625 = 6.25\%$.]

C. 15%
[The candidate thinks the 15 unshaded triangles represent 15%.]

D. 16%
[The candidate thinks that the 16 triangles represent 16%.]

Quantitative Reasoning Example 3

Content Objective: Q4A

A student took five measurements of the freezing point of an unknown chemical solution. The mean of the first four measurements was –8.40 ˚C, and the fifth measurement was –9.00 ˚C. What was the mean of the student's five measurements of the freezing point?

A. –8.88 ˚C
[The candidate finds the temperature that is one-fifth of the way from –9.00 ˚C to –8.40 ˚C, rather than from –8.40 ˚C to –9.00 ˚C.]

B. –8.70 ˚C
[The candidate simply averages –8.40 ˚C and –9.00 ˚C.]

C. –8.55 ˚C
[The candidate finds the temperature that is one-quarter rather than one-fifth of the way from –8.40 ˚C to –9.00 ˚C.]

D. –8.52 ˚C *
[CA: One way to do this is to find the temperature one-fifth of the way from –8.40 ˚C to –9.00 ˚C. Another way is to realize that the sum of the first four measurements (in ˚C) was 4(–8.40), conclude that the sum of all the measurements was 4(–8.4) + (–9.00), and then divide this result by 5 to obtain the mean of all five measurements.]

Quantitative Reasoning Example 4

Content Objective: Q5A2

If $\ln e^{2x+1} = 9$, what is the value of x?

A. $\dfrac{(-1+\ln 9)}{2}$

[The candidate makes a mistake and assumes that $2x + 1 = \ln 9$, then solves the equation and gets $\dfrac{(-1+\ln 9)}{2}$.]

B. 4 *

[CA: The natural logarithmic function and the exponential function are inverse functions. Therefore, $\ln e^{2x+1} = (2x+1) \cdot \ln e = 2x+1$. So, $2x+1 = 9 \Rightarrow 2x = 8 \Rightarrow x = 4$.]

C. $\left[\dfrac{9}{(2)(\ln e)} \right]$

[The candidate makes a mistake and assumes that $\ln e^{2x+1} = 2\ln e$, the solves the equation and gets $\left[\dfrac{9}{(2)(\ln e)} \right]$.]

D. 23

[The candidate makes a computational error.]

Quantitative Reasoning Example 5

Content Objective: Q6C1

If $f(x) = e^{\sqrt{x+1}}$, what is $f'(3)$?

A. $\dfrac{e}{4}$

[The candidate makes a computational error and does not find the correct numerator.]

B. $\dfrac{e^2}{4}$ *

[CA: First we find the derivative:

$$f'(x) = e^{\sqrt{x+1}} \cdot \frac{d}{dx}(\sqrt{x+1}) = e^{\sqrt{x+1}} \cdot \frac{1}{2}(x+1)^{-1/2} = \frac{e^{\sqrt{x+1}}}{2\sqrt{x+1}}$$

Therefore, if we evaluate the derivative at $x = 3$, we have:

$$f'(3) = \frac{e^{\sqrt{3+1}}}{2\sqrt{3+1}} = \frac{e^{\sqrt{4}}}{2\sqrt{4}} = \frac{e^2}{2 \cdot 2} = \frac{e^2}{4}. \text{ Thus, } f'(3) = \frac{e^2}{4}.]$$

C. $\dfrac{e^2}{2}$

[The candidate does not find the derivative correctly and forgets a factor of 2 in the denominator.]

D. e^2

[The candidate does not find the derivative correctly.]

Writing

Writing Prompt Content Objectives
A. Health Issues (issues related to public health, medicine, nutrition, fitness, prevention, treatments, therapies, medications, drugs, attitudes)
B. Science Issues (issues related to research, theories, findings, applications, controversies, education, attitudes)
C. Social, Cultural, or Political Issues (issues related to beliefs, attitudes, behaviors, trends, laws, policies)

Examples of Writing Prompts

- Each writing prompt will be in the form of a statement that presents a contemporary problem relating to one of the general Content Objectives listed above.

- The problem referred to in each prompt may be specific but will also be universal enough that it does not require extensive background in the history, politics, or situation of any particular region of the world.

Writing Prompt Example 1

Content Objective Code: A

Aging populations and growing shortages of organ donor supplies result in thousands of deaths each year of people on waiting lists in the U.S. from such conditions as chronic lung disease, kidney failure, and coronary heart disease. Discuss a solution to the problems resulting from insufficient supplies of donated human organs.

Writing Prompt Example 2

Content Objective Code: A

According to a recent report from the U.S. Census Bureau, people without health insurance are more costly to the health care system, are more likely to delay or forgo medical services, and are generally in poorer health than people with insurance. Discuss a solution to the problem of providing adequate health care for the uninsured.

Writing Prompt Example 3

Content Objective Code: A

According to the U.S. Centers for Disease Control, poor diet and inactivity result in tens of thousands of deaths each year and are major contributors to debilitating conditions, such as diabetes, osteoporosis, obesity, and stroke. Discuss a solution to the problem of promoting healthy dietary habits.

Writing Prompt Example 4

Content Objective Code: B

Even though the use of fossil fuels has enabled worldwide industrial development, it has also been the largest source of carbon dioxide emissions, and the development of renewable resources has been hindered by issues related primarily to high costs. Discuss a solution to the problem of developing affordable alternatives to fossil fuels.

Writing Prompt Example 5

Content Objective Code: B

According to the World Wildlife Federation, as many as one-third of all wildlife species in the United States are at risk of extinction due to decades of habitat loss and the spread of invasive species. Discuss a solution to the problem of protecting endangered wildlife species in a rapidly changing world.

Writing Prompt Example 6

Content Objective Code: B

According to the U.S. Environmental Protection Agency, an average temperature increase of nearly 1.5°F over the past century and projections of an even greater rise over the next century could result in dangerous climate and weather changes with challenging social and environmental consequences. Discuss a solution to the problem of of dealing with global warming.

Writing Prompt Example 7

Content Objective Code: C

A functioning democracy requires well-informed citizens who actively participate in voting and other civic activities, but research suggests that such forms of civic engagement are declining among people in the United States today. Discuss a solution to the problems resulting from a lack of participation in civic life.

Writing Prompt Example 8

Content Objective Code: C

Immigration policies are being debated in the United States and other countries, especially where many immigrants are illegal, assimilation is challenging, unemployment is relatively high, and needs for both skilled and unskilled workers are not met by citizens. Discuss a solution to the problem of establishing immigration policies that are both effective and fair.

Writing Prompt Example 9

Content Objective Code: C

According to the U.S. Department of Justice, the deliberate defacing or destroying of public or private property results in more than $15 billion in damages to schools, businesses, and individuals each year. Discuss a solution to the problem of reducing graffiti and vandalism in neighborhoods where they are common.